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PORTFOLIO TURNOVER AND COMMON STOCK HOLDING PERIODS

In observing the relative performances of common stock portfolios over the years, it has been my impression that the more successful portfolios have had average turnover rates which, over time, have gravitated to about 25% per year which, in turn, has implied average holding periods for the stocks in the portfolios of about four years. Additionally, it is usually the more recently acquired common stocks in such portfolios that seem more appropriate candidates for sale than stocks that have been in the portfolios for longer periods of time. The purpose of this paper is to try to incorporate some bases in logic for these two empirically inferred (and perhaps counter-intuitive) findings.

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PORTFOLIO TURNOVER DEFINED

Turnover is defined as the ratio of the total of all purchases in a portfolio over some period of time to the average value of the portfolio over that period of time. The Looper formula, as it is commonly known, is expressed as follows:

$$\text{Portfolio Turnover} = \frac{\text{Total Purchases}}{\text{Average Portfolio Value}}$$

The period of time used as a reference is usually one year. If the period for which the computations are made is not one year, the number is usually annualized to facilitate comparisons. The Looper formula may, then, be embellished as follows:

$$\text{Average Annual Portfolio Turnover} = \frac{\text{Total Purchases}}{\text{Average Portfolio Value}} \times \frac{365}{\text{Number of Days in Period}}$$

With increasing precision, "Average Portfolio Value" may be the beginning <u>or</u> ending value of the portfolio for the period, the average of the beginning <u>and</u> ending values, the average monthly values, the average weekly values, or the average daily values.

The turnover figure calculated is also far more meaningful if the period covered is several years, rather than just several months. In fact, if the period is too short, the turnover figure will be meaningless. As an example, if somebody creates a common stock portfolio by investing the proceeds of a maturing certificate of deposit in common stocks and decides to measure his portfolio turnover with the foregoing formula after one week of ownership, he will come up with an Average Portfolio Turnover of 5,214%, indicating that he buys and sells all the stocks in his portfolio 52 times a year when, in fact, it may be his intention never to sell any of the stocks he has just purchased.

Calculation of Average Holding Period

The concept of "average holding period" is perhaps more easily visualized than "average turnover rate." Average holding period tells us, on average, how long after the portfolio manager purchases a security, he sells it. Fortunately, given either average turnover rate or average holding period, one can calculate the other. Given average turnover rate, the formula for average holding period is as follows:

$$\text{Average Holding Period (in months)} = \frac{\text{12 months}}{\text{Average Annual Turnover Rate}}$$

Various turnover rates, then, generate average holding periods as follows:

Average Annual Turnover Rate	Average Holding Period
5%	20 years
10%	10 years
25%	4 years
50%	2 years
75%	16 months
100%	12 months
150%	8 months
200%	6 months
300%	4 months
400%	3 months
600%	2 months

Implied Average Turnover Rates and Average Holding Periods

There are two major difficulties encountered in trying to calculate turnover rates and holding periods from historical purchase and sale and portfolio evaluation data. The first involves adjustments for major inflows of cash into the portfolio or outflows from the portfolio. If the

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inflows and/or outflows are of significant size and/or frequency, the mathematics become unwieldy. The second difficulty involves the ability to preserve, retrieve, and incorporate into the calculations all the relevant historical portfolio transactions, even if there have been no major cash inflows or outflows.

Fortunately, there is an alternative for estimating these two portfolio characteristics which depends solely upon a static analysis of the portfolio at any given point in time. If one asks the computer to provide a weighted average holding period of all the securities in a portfolio, one has half the battle fought. As long as the portfolio data base includes the date of purchase of each security in it, using amounts owned and current prices, an implied average annual holding period is easily computed. Given the average annual holding period, calculation of the average turnover rate is quite a simple matter, as follows:

$$\text{Average Turnover Rate (in years)} = \frac{365}{\text{Weighted Average Holding Period (in days)}}$$

As alluded to above, using this method, or any other method, a recently created or drastically modified portfolio may not begin to reveal its normal average turnover rate and normal average holding period until the passage of a time interval equal, at least, to whatever that average holding period happens to be.

UNACCEPTABLE RATES OF PORTFOLIO TURNOVER

I find the subject of portfolio turnover an interesting one, in part because of the broad spectrum of numbers among stock market strategists as to what "optimum" turnover might be. Let us, however, begin with what it is pretty much universally accepted optimum turnover is not.

"Churning" is the word used to describe excessive trading, sometimes encouraged by a security salesman to generate excessive commissions. Churning, by definition, then, is a level of portfolio turnover which, at least from the point of view of the portfolio owner, is decidedly greater than optimal. I find the subject of "churning" particularly amusing because of the extremely high rates frequently practiced and also because of the extremely high rates frequently construed as acceptable in courts of law and arbitration proceedings.

Generally, a turnover rate of six times per year (holding each of the securities in a portfolio, on average, for only two months) is regarded as prima facie evidence of churning. A turnover rate of 2½ times per year (an average holding period of 4.8 months) is apt to be the threshold of the definition of churning in an arbitration proceeding.

Back in the 1960s, a writer in the *Harvard Law Review* ranked turnover rates, based on the Looper calculation, in what has become known as the "2-4-6" formula. This often-used rule-of-thumb is defined as follows:

Average Turnover	Average Holding Period	Degree of Indication of Excessive Turnover
200%	6 months	Inferential
400%	3 months	Presumptive
600%	2 months	Conclusive

A series of classic court cases covering the four decades following World War II has also indicated a general acceptance of surprisingly high rates of portfolio turnover. As seen in the tabulation below, in fifteen cases in which the turnover rates were construed as excessive, the average holding period ranged from as short a period as four days to as long a period as sixteen months, with an average of one month and a median of four months. Similarly, in the seven cases in which the turnover rate was deemed acceptable, the average holding period ranged from as long as fifteen months to as short as two weeks, with an average of two months and a median of six months.

Excessive Rates of Portfolio Turnover						Acceptable Rates of Portfolio Turnover		
Year	Turnover	Avg Holding Period	Year	Turnover	Avg Holding Period	Year	Turnover	Avg Holding Period
1947	150%	8.0 months	1980	200%	6.0 months	1953	2,500%	2.1 weeks
1962	158%	7.6 months	1982	600%	2.0 months	1975	338%	3.6 months
1964	293%	4.1 months	1984	667%	1.8 months	1976	80%	15.0 months
1965	327%	3.7 months	1984	2,600%	2.0 weeks	1977	700%	1.7 months
1965	8,939%	4.1 days	1985	893%	1.3 months	1978	185%	6.5 months
1968	143%	8.4 months	1985	1,202%	1.0 month	1984	187%	6.4 months
1968	200%	6.0 months	**Average**	**1,106%**	**1.1 month**	1987	200%	6.0 months
1969	77%	15.6 months	**Median**	**293%**	**4.1 months**	**Average**	**599%**	**2.0 months**
1970	143%	8.4 months				**Median**	**200%**	**6.0 months**

Turnover Rates Among Institutional Investors

The average turnover rates among the nation's professionally managed pension funds is said to be about 70%, indicating an average holding period of 17 months. Because mutual funds operate in a fish bowl, because there are so many of them, and because their operations are so exhaustively studied, however, it is probably these institutional investors that provide the best sampling of the level of trading activity among the nation's professionally managed institutional portfolios.

In this regard, in 1998, the 435 mutual funds categorized by *Morningstar* as "Large-cap Growth" funds had an average turnover of 93% (a 12.9-month average holding period), the 195 funds categorized as "Mid-cap Growth" had an average turnover of 108% (an 11.1-month holding period), and the 183 funds categorized as "Small-cap Growth" had an average turnover of 120% (a 10-month holding period). Over the ten-year period 1989-1998, the "large-cap growth" funds had average turnover rates of 93% (12.9 months), and both the mid-cap and small-cap growth funds had average turnover rates of 114% (10.5 months).

Of equal fascination is the extraordinary rates of turnover of the more active mutual funds. The twenty-five most active growth funds covered by *Morningstar* in 1998 had portfolio turnover rates that ranged from 215% to 972% and averaged 320%, which rates translate into average holding periods of 24 weeks, 5 weeks, and 16 weeks, respectively

Incidentally, it is, to a large extent, the high turnover rates characteristic of mutual funds that is responsible for their annual total returns' averaging significantly less than benchmark indices used to measure the performances of the particular market sectors in which they invest. High turnover rates exacerbate the problem, unique to large institutional investors such as mutual funds, known as "market impact costs"—the costs, over and above the usual operating expenses and marketing (12b-1) fees, associated with the sacrifices in price that must be incurred when trading large blocks of stock.

TURNOVER RATES IN MUTUAL FUND BOND PORTFOLIOS

Though our primary interest here is the management of common stock portfolios, my most stunning discovery in researching for this paper was the extraordinarily high rates of turnover that prevail in the portfolios of mutual funds that invest solely in high-quality bonds.

Traditional investing assumes that high-quality bonds are purchased to be held to maturity, in which case the turnover in such a bond portfolio should be quite minimal. If we buy equal amounts of bonds each year with maturities of five years and hold them to maturity, our average rate of turnover will be 20%; if we buy ten-year maturities, our turnover rate will be 10%; and, if we buy twenty-year maturities, our turnover rate will be 5%.

Overwhelmingly, the prime determinants of the value of high-quality bond portfolios are changes in the level and structure of interest rates. Therefore, in the case of a high-quality bond portfolio, the only justification for active management is the belief that the portfolio manager can forecast changes in interest rates, and buy and sell bonds in accordance with his forecasts, with enough reliability to outperform a "buy-and-hold strategy" and by a margin great enough more than to cover the cost of retaining his services. (Junk bond portfolios might be expected to be more actively managed than high-quality bond portfolios, since changes in the fortunes of the underlying company impact the <u>safety</u> of a junk bond. In such a case a change in the <u>quality</u> of

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the bond, as well as changes in interest rates, may be a major determinant of changes in its value.)

The performance data on actively managed high-quality bond portfolios is not encouraging, however. The following tabulation is insightful:

	LONG-TERM HIGH-QUALITY CORPORATE BONDS	LONG-TERM U. S. GOVERNMENT & AGENCY BONDS
1998		
Number of Funds in Composite	55	33
Average Turnover	163%	168%
Average Holding Period	7.4 months	6.1 months
Operating Expenses (Expense Ratio)	1.06%	1.10%
Total Return Shortfall Relative to Index	-5.37%	-3.49%
1989-1998		
Number of Funds in Composite	18 in 1989 to 55 in 1998	19 in 1989 to 33 in 1998
Average Turnover	139%	170%
Average Holding Period	8.6 months	7.0 months
Average Annual Operating Expenses	1.00% per year	0.89% per year
Average Total Return Shortfall Relative to Index	-2.24% per year	-1.78% per year

For corporate bonds the benchmark index is the Lehman Brothers Corporate Bond Index. For U. S. Government bonds, the benchmark index is the Lehman Brothers Long-Term Government/Corporate Bond Index. The performance of an index is generally accepted as the equivalent of the performance of a randomly selected and unmanaged portfolio of the securities in the particular market sector being measured. It is, therefore, the equivalent of a "buy-and-hold" investment strategy.

Remarkably, mutual funds that invest in high-quality bonds, on average, are actually more actively traded than are mutual funds that invest in common stocks.

As seen in the foregoing data, the significant amounts by which the underperformance of high-quality mutual fund bond portfolios exceeds their average annual operating expenses is clear proof that the return on their high rates of activity is negative.

IS THERE PROBABLY AN OPTIMUM RATE OF PORTFOLIO TURNOVER?

Other than my own, I am aware of no empirical studies designed specifically to determine optimum rates of portfolio turnover. Furthermore, I would be reluctant to subject my own studies to tests of scientific rigor. In fact, based upon my own observations alone, I am more comfortable calling my conclusion that the magic number is 25% (implying an optimum average holding period of 4 years), more of a "hunch" than a demonstrable fact.

Before trying to defend these 25% and 4-year figures, however, let us examine the proposition that there may even be any validity to the concept of an "optimum" rate of portfolio turnover or "optimum" average holding period for a common stock.

Let us assume that there is a publicly traded company scheduled to report its earnings tomorrow and that it is generally accepted, as a near certainty, that the company will announce an earnings increase of 50%. Should we purchase that stock today with the expectation of being able to sell it tomorrow at a profit brought about by the actual announcement of the 50% increase in earnings? Intuitively, we all know that this would not be a good reason for buying the stock. But why would it not be a good reason for buying the stock?

The explanation lies in the foregoing phrase "generally accepted." It is "generally accepted" that earnings will be up 50%. Everybody who has inquired believes earnings are going to be up 50%. Hence, the 50% earnings increase is already factored into the price of the stock. To put it into more technical jargon, the price of the stock today already "discounts" the earnings increase to be announced tomorrow. There will be no more profit left to be made in the stock tomorrow as a result of the earnings announcement. The stock has already risen to reflect tomorrow's inevitable earnings announcement.

If we know about the big earnings increase to be announced tomorrow, but nobody else knows about it, we have a different situation. We can probably buy the stock (or, still better, buy call options on the stock) today and sell tomorrow and make an enormous profit. In such a case, however, we are "insiders" with "nonpublic information" and so, if we do act on such information, we must also consider going to jail as one of the likely outcomes.

Let us next consider a company which we have studied with great care and conclude that, because of some unique product or service it provides, it should increase its sales and profits a hundred-fold over the next ten or twenty years, in which case the price of the stock had also ought to go up a hundred-fold over that period of time. We believe it will be another Microsoft or Wal-Mart. Why should we not sell all of our other financial assets and mortgage our house and put every last dime we can dig up into this promising company?

Again, our intuition, if not our experience, tells us that the time frame is too long to ensure accuracy in our prediction. We know that we can use Microsoft and Wal-Mart as examples only with the benefit of hindsight. When those companies were in their infancies, their prospects looked no better than did those of hundreds of other companies much like them. To have been confident of purchasing a Microsoft or a Wal-Mart in their infancies we would have had to purchase ninety-nine other companies that looked just like them at the same time, but which subsequently did not make the grade. With only one one-hundredth of our investment in the big winners, our overall results over the ten- or twenty-year period would have only mirrored the "aggressive growth" stock sector of the stock market, even though Microsoft and/or Wal-Mart were included among our holdings.

Clearly, if the period of time over which we predict is too short (days), the effects we predict are already incorporated, or discounted, in the price of the stock, and so we cannot make above-average profits by acting upon those predictions, even though our predictions are quite accurate. Similarly, if the period of time over which we predict is too long (decades), the competitive dynamics and uncertainties of capitalism make such predictions extremely unreliable, and so we cannot make above-average profits by acting upon those predictions either.

The implication would seem to be that, if there is some reasonable or optimum average period over which judgments about individual common stock can be made, it is a period so long as to be measured in units longer than days, but also a period not so long as to be measured in decades. To describe this period of time, let us coin the phrase "Optimum Period of Prediction"

What Might Be the Length of the "Optimum Period of Prediction" in the Management of a Common Stock Portfolio?

In reviewing the literature of common stock, portfolio, and market analyses, one is bound to be impressed by the frequency with which four-year cycles and four-year time horizons are encountered.

Though the divergences have been very wide, the stock market itself is said to have a "natural" cycle of 48 months. The business cycle, too, over very long periods of time, has averaged just about four years. What the Federal Reserve Bank does in controlling the money supply appears to have a lag time of four years before its impact is felt on the rate of inflation. These four-year cycles are frequently regarded as being influenced by the four-year presidential election cycle.

Many analysts use three-to-five year periods (the mid-point of which, of course, is four years) over which they attempt to project a company's earnings. *Value Line*, in particular, uses time frames of three-to-five years in making its longer term projections. *Value Line* has further demonstrated that its composites of three-to-five year appreciation potentials for individual stocks has been amazingly reliable in predicting major moves in the stock market as a whole, four years later.

The Theory of Chaos

The most compelling studies that I have encountered in support of 25% turnover rates and 4-year holding periods have been conducted by a mathematician by the name of Edgar E. Peters. In addition to being a student of mathematics, Peters is a classically trained economist who studied

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under Nobel Laureate Harry Markowitz, the father of modern portfolio theory. Peters has published two books—*Chaos and Order in Capital Markets* and *Fractal Market Analysis: Applying Chaos Theory to Investment & Economics*—not surprisingly, melding his interests in mathematics with the world of investing.

Not only does the word "chaos" appear in the titles of both of Peters' books, but the concept of chaos underlies his theories of the way the securities markets behave. For this reason, let us grapple with the term "chaos" herewith. The philosopher George Santayana defines chaos as "any order that produces confusion in our minds." As mathematicians define chaos, mental confusion may be an outcome, but it is not its essence. A more technical definition says that chaos is

> a deterministic nonlinear dynamic system, with fractal characteristics and a sensitive dependence on initial conditions, that can produce random-looking results.

In an effort to impart meaning to such jargon, let us talk about it in terms of the stock market. In fact, let us talk about it in terms of the hypothetical analysis of a single stock.

In a "deterministic dynamical system," given perfect knowledge of the initial conditions, the future is perfectly predictable. It is the famous mathematician, Pierre Laplace, to whom is generally attributed original exposition of the doctrine that, given precise knowledge of the initial conditions, it should be possible to predict the future of the entire universe.

Presumably, if we have perfect knowledge about the current status of Company A and its common stock—which includes perfect knowledge about all the factors that will affect the company and its stock, both internally and externally, and the relationships among those factors—we can know all we need to know to predict the future of Company A, including the future price of its common stock. We can create a mathematical model whereby we input the initial conditions (our company analysis), and our model identifies the state of our company at any future time we specify.

The characteristics of a dynamical system that make it "chaotic" are the presence of a "large set" of initial conditions which are highly "unstable" and the system's "sensitive dependence upon" these initial conditions. The terms "large set" and "unstable" would seem to describe appropriately the number and character of the variables we would encounter if we were to try to list all of the factors, both internal and external, that completely describe Company A, its operating environment, and the price of its stock, as we study it today.

It has been suggested that the concept of "sensitivity to initial conditions" may be understood by imagining a boulder precariously perched on the top of a hill. The slightest push will cause the boulder to roll down one side of the hill or the other. The subsequent behavior of the boulder depends upon its sensitivity to the direction of a push—the magnitude of which push may be quite small. If we are located at the bottom of one side of the hill, we are keenly interested in which direction the boulder will be pushed. In a chaotic deterministic dynamical system, all, most, many, or at least some of the initial conditions are like boulders precariously perched on the tops of hills.

A system of chaos is often described as a non-linear system. The difference between a linear system and a non-linear system is that a non-linear system relates the variables on either side of the equation with powers other than one. Probably the simplest illustration comes from our high school algebra and geometry. As seen in the following table, the relationship between the circumference of a circle and its radius is linear. The relationship between the area of a circle and its radius is non-linear, however, because the radius of the circle must be squared (carried to the 2nd power) to get the area. Similarly, the relationship between the volume of a sphere and its radius is non-linear because the radius of the sphere must be cubed (carried to the 3rd power) to get its volume.

			Underestimate		Overestimate	
Variable	**Formula**	**Actual**	**Estimated**	**% Error**	**Estimated**	**% Error**
Radius	$r =$	10.00 inches	9.00 inches	-10%	11.00 inches	+10%
Circumference of Circle	$2\pi r =$	62.83 inches	56.55 inches	-10%	69.12 inches	+10%
Area of Circle	$\pi r^2 =$	314.16 sq. in.	254.47 sq. in.	-19%	380.13 sq. in.	+21%
Volume of Sphere	$\frac{4}{3}\pi r^3 =$	4,188.79 cu. in.	3,053.64 cu. in.	-27%	5,575.29 cu. in.	+33%

Notice, also, in the foregoing table that, if we make a 10% error in measuring the radius of a circle, we shall have a 10% error when we calculate its circumference. This is a linear relationship. If we try to measure the area of a circle with a 10% error in our measurement of its radius, however, we end up with an error of 19% to 21% in the area. And, if we try to measure the volume of a cube with a 10% error in our measurement of its radius, we come up with an error of 27% to 33% in our volume.

It is, then, this non-linearity of so much of the real world that makes it so hard to construct mathematical models with which to predict with a very high degree of accuracy. Imagine the price of the stock of our Company A related to hundreds or thousands of variables by powers far in excess of one, two, or three.

The way mathematical chaos manifests itself is by the observation that, no matter how precisely we measure the initial conditions in a system (study the company), our prediction of its subsequent behavior can go radically wrong after a short period of time. Errors in our initial measurements compound themselves over time at an "exponential" rate; or, put another way, the horizon of predictability of such a system grows "logarithmically" with the precision of measurement. What the latter means is that, while we may increase the precision of our initial measurements (our company analysis) by ten-fold, the reliability of our predictions may increase at some much lesser rate—by only two-fold, for example.

In spite of the fact that there appear to be so many complex relationships that determine the nature of the world around us, the predictive sciences are not all lost causes. As we watch weather forecasters try to predict the path of a hurricane through the Caribbean and into the Gulf of Mexico or up the East Coast, we appreciate how much more confident they are about their predictions for the coming day than they are about their predictions for the coming week.

Depending upon the complexity of what we are trying to predict and the use to which we want to put our predictions, there is probably some time frame over which our predictions can be put to good use, even in chaotic systems.

Though our everyday use of the term might suggest otherwise, mathematical chaos is definitely not complete disorder. It is a level of disorder whereby predictions may be made with some degree of reliability, though not over the very long-term. This would appear to be the explanation of the apparent utility of price and earnings "momentum" stock market strategies that work over shorter periods of time, but not for the long-term.

Chaos theory seems to govern stock market investing somewhat as it governs the growth of an oak tree. We can plant an acorn with a high degree of confidence that an oak tree will grow, but we still have little idea of exactly what the oak tree ultimately will look like. With respect to the volatility of the stock market, chaos theory explains why we might be correct about what will happen in the future, without having the foggiest idea of when it will happen or how severe the happening will be. Major events, like stock market crashes, can be expected, but they cannot be predicted.

What mathematical chaos, as applied to the analysis of common stocks, seems to do for us is provide a conceptual framework for accepting the notion that, though we have some chance of predicting the behavior of individual common stocks over some limited periods of time, we have virtually no chance of making such predictions reliably over very long periods of time. Though individual common stocks may appear to behave in a random fashion over very long periods, they may exhibit discernible patterns over shorter periods.

Mathematical chaos is not an attribute of common stock investing alone. It has application to most of the world's more complex natural phenomena. Systems of chaos are used to describe the nature of biological evolution; they are used in chemistry, physics, medicine, engineering, economics, and even in forecasting the weather. An American meteorologist, Edward Lorenz, in attempting to replicate a calculation in his studies of the weather, discovered that simply

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rounding his initial conditions to three decimal places rapidly led to widely divergent results. He concluded, therefrom, what has become a classic analogy called the "butterfly effect": the mere flapping of a butterfly's wings in Brazil, Lorenz said, may set off a tornado in Texas.

In his book, *Chaos: Making a New Science*, James Gleick writes, "The most passionate advocates of the new science go so far as to say that twentieth century science will be remembered for just three things: relativity, quantum mechanics, and chaos." Each of these sciences is primarily interested in understanding reality at a characteristic scale: quantum mechanics works at subatomic dimensions; relativity, at the galactic scale where speeds approach the upper limit of light; and chaos theory, at the scale of everyday life.

FRACTALS

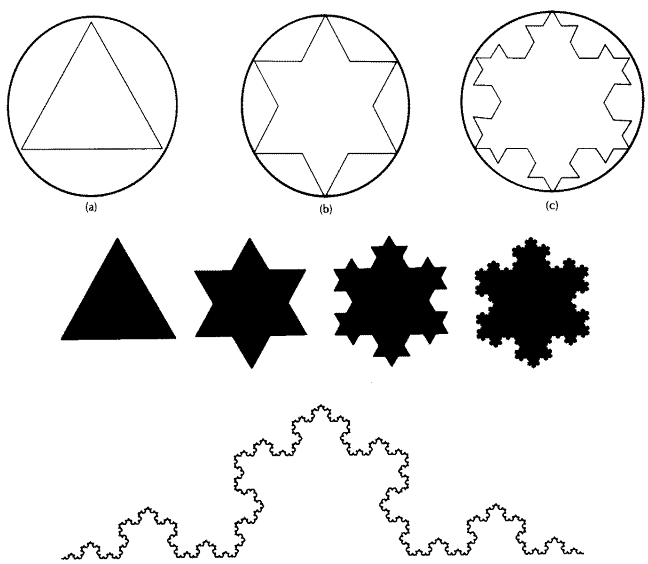
Useful to the understanding of the theory of chaos and its application to the stock market is an understanding of "fractals."

A fractal is an object, a system, or a process for which the parts are in some way related to the whole; that is, the individual components are said to be "self-referential" or "self-similar." An example is the branching network in a tree. While each branch and each successive smaller branch is different, all the branches are qualitatively similar to the structure of the tree as a whole.

The science of fractals is frequently illustrated with what are known as "geometric" fractals, the best-known of which are the "Koch Snowflake" and the "Sierpinski Triangle." Let us examine each:

The Koch Snowflake appears below. It is constructed according to the following rules: (a) Construct an equilateral triangle. (b) Add three new triangles, extending outward, with the middle third of each side of the first triangle as the base of each new triangle. (c) Continue, indefinitely, to add new equilateral triangles to the middle third of each side of each new triangle, extending outward in the same way.

If we continue with the reiterative process described above long enough, we eventually come up with a snowflake-like object, a magnified portion of which appears as the last of the above illustrations. Incidentally, though this process may be repeated an infinite number of times, no part of the snowflake’s perimeter ever falls outside a circle drawn through the three vertices of the original triangle.



For our purposes here, the important observations are that a simple formula is used to describe a process for modifying a simple structure, and this process may be repeated an indefinite number of times to arrive at a much more complex structure. Each subsequent version represents simply a propagation of earlier versions down to a smaller scale. Most important, the instructions for constructing the last infinitesimally small triangle are exactly the same as for constructing the first three triangles in illustration (b) above. The "genetic code" for the entire structure, which eventually consists of an infinite number of infinitesimally short straight lines, is implicit in the code for creating the first three appended triangles. The process, from beginning to end, may be said to demonstrate a "long memory" for its "initial conditions."

The Sierpinski Triangle is constructed as follows: (a) Start with a solid equilateral triangle. (b) Remove an equilateral triangle from the center of the first triangle. (c) Remove equilateral

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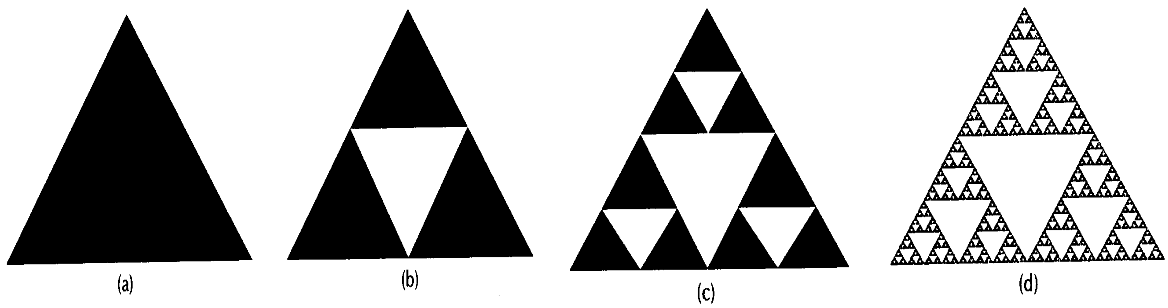
triangles from the remaining triangles. (d) Repeat, indefinitely, removing a triangle from each newly created triangle.

As with the Koch Snowflake, the Sierpinski Triangle, a complicated structure is created via the reiteration of a very simple rule; in every stage of the figure's evolution, the basic structure of all



the stages that came before is retained—the first stage, and every stage thereafter, contains the blueprint or genetic code for all the stages that follow. Again, the process manifests a "long memory" for its "initial conditions."

The "memories" of geometric fractals remind us of many of the processes we see in nature. The fractal structure and growth of a tree has already been mentioned. The human vascular system, with its complicated assemblage of arteries and veins down to capillaries so small that they will pass no more than a molecule of blood at a time, provide another example. The propagation of a species also illustrates the principle. Presumably, the dominant characteristics of those of us alive today, and those to be born tomorrow, were inherent in the genetic code of our ancestors who lived thousands of years ago. Mother Nature seems to have a very long memory for her initial conditions, irrespective of when we might select "initial" to have been.

Fractal objects, systems, and processes are said to be "different in detail but similar in concept." More technically, they are said to be "locally random, but globally ordered or deterministic."

TIME SERIES AND THE CAPITAL MARKETS

A time series is simply a graph of the behavior of some variable over time. If we plot the average temperature or the range of temperatures for each day for a year, we have a time series. Most stock market charts are time series in that they plot price changes in a stock or a stock market index over some period of time.

Time series fulfill the fractal criteria of being locally random but globally ordered. The randomness of a stock market graph, for example, is described as "noise" and is compared to the static or snow interference we may get with a radio or television transmission. The "signal" or program being transmitted represents the global order.

Because many time series exhibit fractal characteristics, techniques similar to those used to measure the characteristics of geometric fractals are used to measure the fractal characteristics of time series.

The pioneer in this field was a hydrologist by the name of H. E. Hurst. Hurst began working on the Nile River Dam project about 1907 and remained in the Nile region for the next forty or so years. Given widely varying rates of rainfall and water inflow, his problem was to control the discharge rate of the reservoir so that it would neither overflow nor run dry. Hurst developed a technique called "rescaled range analysis" which enabled him to measure the memory in a time series, now referred to as the "Hurst exponent." He found that most natural phenomena, including river discharges, temperatures, rainfall, and sunspots, follow a pattern described as a "biased random walk"—a trend with noise.

PETERS' APPLICATION OF CHAOS THEORY TO COMMON STOCK CYCLES

Edgar Peters' contribution has been to extend to the capital markets the rescaled range analysis techniques which Hurst applied to natural phenomena.

It is perhaps useful to begin our summary of the work of Edgar Peters with a definition of the word "cycle" as it us used in the theory of chaos.

We usually think of a cycle, such as the cycle of day and night, as being defined by returns to an initial state (peak-to-peak or trough-to-trough), periodically over identical durations of time. If our daily cycle begins at noon today, it is complete at noon tomorrow, exactly twenty-four hours later. Cycles in the theory of chaos, however, are bound by neither constraint. There need not be a return to an earlier state, nor need a cycle be periodic. A cycle in chaos theory is defined simply as a change in direction. The economy will expand for some indeterminate period, and then it will contract for another unknown period. It will, however, rarely contract exactly to its size before the previous expansion began, nor are business cycles of uniform duration. These expansions and contractions are called cycles, nevertheless. Chaotic cycles are nonperiodic in that their time components cannot be individually determined in advance. A cycle is better visualized here as a "measure of persistence" or the "duration of a trend." In the discussion of the capital markets, a cycle is a "statistical" cycle which measures the length of time over which information impacts a market.

The Hurst exponent can vary between 0.0 and 1.0. 0.5 represents a purely random or utterly unpredictable time series. Hurst exponents of less than 0.5 indicate the presence of what is known as "antipersistent" behavior, while Hurst exponents greater than 0.5 indicate the presence of a long-term memory of previous conditions. Most of the capital markets exhibit Hurst exponents that are greater than 0.5.

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With a Hurst exponent greater than 0.5, more recent events have a greater impact than events more distant in time, but the latter still have residual effects. Today's events ripple forward in time like the ripples from a pebble dropped in water. A ripple may persist for quite some time and distance, but it diminishes steadily until, for all intents and purposes, it finally vanishes.

The Hurst model, as applied to the capital markets, implies that, at any given point in time, a set of economic conditions creates a bias in a company's performance, and that this bias persists until the random arrival of some new and significant information that changes the bias in magnitude, direction, or both.

Using the "rescaled range analysis" technique of chaos theory, and using the Standard & Poor's 500 data covering the 62-year period from 1928 to 1989, as well as the record of the Dow-Jones Industrials for the 102-year period between 1888 and 1990, Peters has demonstrated that the stocks in the U. S. stock market do, indeed, have average cycles of approximately 48 months. What Peters means is that the price of a common stock appears to have a memory of its initial conditions that lasts for 48 months. The parameters that define a company's condition today will continue to affect that company for approximately 48 months. The price of the stock will continue to be biased by the dynamics of its initial state for 48 months.

It is also interesting to note, however, that Peters found that certain sectors of the market had different cycles. Cycles for electric utilities extended out to six to seven-and-one-half years. Industrial companies tended to have cycles that averaged somewhat less than 48 months, while high-technology stocks, in particular, had cycles that averaged only eighteen months. Industries characterized by higher rates of innovation appeared to have shorter natural cycles. His findings for some specific companies are summarized below:

STOCK	LONG-TERM MEMORY (MONTHS)
Apple Computer	18
IBM	18
Xerox	18
Coca-Cola	42
McDonald's	42
Anheuser-Busch	48
Niagara Mohawk Power	72
Consolidated Edison	90
Texas State Utilities	90

Another interesting observation that Peters made regarding the behavior of stock market prices is that, if one tries to measure memory using increments of time less than 30 days, noise overwhelms signal. The implication is that discussion about a stock's price fluctuations, from day-to-day, or even from week-to-week, is not likely to be meaningful. It is not until after we have a series of data that can be measured in months that we can detect in the data a signal sufficiently strong to be heard over the noise, or seen through the snow, to enable us to make enlightened inferences about a common stock's performance.

AN INTERPRETATION OF NATURAL COMMON STOCK CYCLES AS A GUIDE TO ARRIVING AT OPTIMUM PORTFOLIO TURNOVER RATES

The implication of the above-described phenomena is that the major forces that typically impact industries and companies and the biases that influence the prices of their common stocks tend to persist over periods of time that average four years. It implies that these forces have not only an immediate effect but have a lingering effect as well which lasts, on average, about four years.

It should not be surprising, then, if one observes that the stocks in the most successfully managed portfolios appear to have average holding periods of about four years which, in turn, means average rates of portfolio turnover of the order of 25%.

In fact, for portfolios minimally invested in utilities and/or with an emphasis on higher technology companies, somewhat shorter average holding periods and somewhat higher rates of portfolio turnover are to be expected.

In an article by Robert H. Jeffrey and Robert D. Arnott, in the Spring 1993 issue of the *Journal of Portfolio Management*, I find the following:

> Since any sensible investor understands that a buy-and-hold strategy, if pursued long enough, must inevitably result in flat and eventually negative growth as the holdings mature, portfolios must therefore be pruned, and pruning means turnover, which means realizing gains... [C]onventional wisdom thinks of any turnover in the range of, say, 1% to 25% as categorically low...and of anything greater than 50% as being high...

I have personally come to be quite comfortable with such a perception as a part of my own investment philosophy.

THE LIFO PHENOMENON IN PORTFOLIO MANAGEMENT

LIFO and FIFO are acronyms, respectively, for "Last In, First Out" and "First In, First Out" inventory accounting. It has been my observation that, if one analyzes a portfolio of common stocks in an objective fashion, based upon the fundamentals of the underlying companies, one will conclude that a greater-than-random portion of the common stocks more recently acquired will appear to be more logical candidates for sale than those common stocks held in the portfolio for longer periods of time. In other words, LIFO seems to describe typical portfolio turnover better than FIFO.

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Whether via intuition or the application of chaos theory, one might expect companies held for longer periods of time to have more likely matured, or to have encountered problems not foreseen at the time of original purchase, than companies more recently acquired. In fact, if the time between the recommendation to purchase a stock and the recommendation to sell it is too short, there is an understandable implication that the one making the original recommendation did not do his homework well.

In a taxable account, the bias toward selling more recently acquired stocks is easier to understand. Stocks held for a long time are more apt to have large capital gains by virtue of the passage of time alone, and so a large tax cost associated with their sale. Stocks recently acquired, on the other hand, have had less time to accrue significant gains and so are less apt to have significant adverse tax consequences if sold. Furthermore, if a stock is sold at a loss in a taxable account, Uncle Sam will subsidize the sale. In short, in a taxable account, given a group of stocks for which the quality, prospects, and position sizes are all considered equivalent, the least attractive candidate for sale will be the issue with the highest percentage gain, while the most attractive candidate for sale will be the issue with the biggest percentage loss. The odds are very great that the stocks with the lower percentage gains or larger percentage losses will have been more recently acquired than the stocks with the larger percentage gains. Tax considerations, then, do explain much of the LIFO turnover bias in a taxable account.

Nevertheless, even in nontaxable accounts—IRAs, pension accounts, and charitable organizations—the LIFO phenomenon still prevails. An objective review of such an account will still usually show that the least desirable holdings are biased toward the issues more recently acquired. This is a paradox.

BUY, HOLD & SELL CATEGORIES

To help understand this LIFO phenomenon in portfolio management, it is useful to recognize that most portfolio managers put securities into one of three categories: (1) "buys"—issues so attractive that their purchase is indicated, if they are not already owned; (2) "holds"—issues not attractive enough to buy, but attractive enough to retain, if currently owned; and (3) "sells"—issues deemed so unattractive as to warrant their disposal.

The LIFO phenomenon is a paradox because the expected evolution of a common stock in a portfolio is from a "buy," to a "hold," to a "sell." At any given time, most of the issues in a portfolio will be classified as "holds."

The difference between a company classified as a "hold" and a company classified as a "buy" is that, while the former is enjoying moderate growth, the latter is in a more innovative, and dynamic, and, so, fragile stage of growth. Key words here are "innovative" and "fragile."

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For example, while we may continue to hold a company that is showing earnings growth of 5% to 10% per year, we are apt to require earnings growth of 10% to 20%, or more, before we consider a company a candidate for purchase. The faster growing company is probably currently more innovative and participating in a market that is changing more dynamically and certainly one that is attracting more competition. Because such a company's endeavors are characterized by higher risk, it is more apt to experience a severe relative reversal of fortunes than is a company plugging along at the slower rate of growth. In short, the faster-growing company we recently acquired is more apt to have stumbled and so surfaced as a "mistake" than is the slower-growth company we had simply continued to hold.

In an effort to make this concept more vivid, imagine that today we review a four-stock portfolio and conclude that two companies should be held and two should be sold and replaced by two others. The two that should be sold are no longer growing. The two that are to be held are growing at 10% per year, while the two we want to buy are growing at 20% per year.

Though we will not know it until after the fact, the two stocks to be held, from this point forward, will have an average future life in the portfolio of four years. One will have three years and the other will have five years. The two new stocks we acquire will also have an average life in the portfolio of four years; but, in this case, one will be one year and the other will be seven years. If we review the portfolio one year hence, it will, therefore, be the one of the two stocks acquired just one year previous that will be the candidate for sale.

Though each pair of stocks—the two "holds" that are growing at 10% per year, and the two "buys" that are growing at 20% per year—have average future life expectancies in the portfolio of four years, the "dispersion" around that average is greater for the faster growing companies. In other words, with respect to the individual companies, our expectations are apt to be wider from the mark with the fast-growing companies than with the slower growing companies. We may be as apt to err on the low side as on the high side, but our potential for error is decidedly greater with the faster growing companies.

Furthermore, in terms of the "Hurst exponents" and memory cycles discussed above, a company in an innovative stage of its evolution is apt to have a shorter memory for current conditions than a less innovative company, or even the same company in a less innovative stage of its development. In other words, our "buys," because they represent companies in more innovative periods of their development, may be expected to have shorter memories for the current conditions under which they are bought than other companies in the portfolio currently classified as "holds."

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CONCLUSION

In summary, it appears to me that the most successful common stock portfolios, after the passage of several years following their creation or restructuring, have turnover rates that average about 25% per year, implying average holding periods for the individual stocks in such portfolios of about four years.

Taxable portfolios with large unrealized capital gains may have average turnover rates of somewhat less than 25%, while nontaxable portfolios and portfolios emphasizing more dynamically growing companies in industries characterized by higher rates of innovation may have average turnover rates somewhat in excess of 25%.

Though these concepts of holding periods and turnover rates are useful in the aggregate, when dealing with an entire portfolio over an extended period of time, they are relatively useless concepts when examining a single common stock or a single transaction. Just as one would learn little about an airline's record of delayed departures by examining the data on just one flight, it is necessary to examine the average turnover rate and average holding period for an entire portfolio over some reasonable period of time before conclusions can be drawn as to whether the portfolio is being neglected or is unduly active. As long as such limitations are recognized, however, data on portfolio turnover and average holding periods can be useful guides to portfolio management.

Finally, it should be expected that more recently acquired stocks are more apt to be candidates for early sale, not only because of tax considerations in a taxable account, but also because of the greater vulnerability of companies to severe reversals of fortune when they are enjoying periods of especially innovative and dynamic growth, as is more apt to be the case at the time of purchase and shortly thereafter.

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